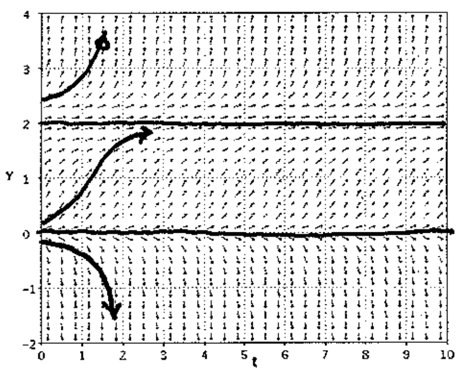
MATH222-004, QUIZ #1, January 27, 2016

NAME: Solution

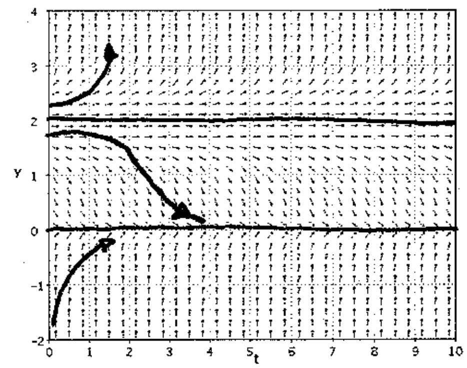
Determine the equilibrium solutions and choose the correct direction field for the differential equation

$$y' = y(y-2)^3$$

Based on the direction field determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency.



(a) Left



(b) Right

Equilibrium solutions: $y(y-2)^3 = 0$ when $\underline{y=0}$ or $\underline{y=2}$.

To decide which picture corresponds to the diff. eqtn. draw equil. solutions and some solutions on them.

For the Left: $y' > 0$ for $y > 2$, $y' > 0$ for $0 < y < 2$, $y' < 0$ for $y < 0$.

for the Right: $y' > 0$ for $y > 2$, $y' < 0$ for $0 < y < 2$, $y' > 0$ for $y < 0$.

For the diff. eqtn.: $y' > 0$ for $y > 2$, $y' < 0$ for $0 < y < 2$, $y' > 0$ for $y < 0$. So the $\underline{\text{Right}}$ figure is the direction field of the diff. eqtn.

Behavior of solution: Initial condition is $y(0) = y_0$.

Thus, if $y_0 > 2$ then $\lim_{t\to\infty} y(t) = \infty$

if $y_0 = 2$ then $\lim_{t\to\infty} y(t) = 2$

if $y_0 \in (0,2)$ then $\lim_{t\to\infty} y(t) = 0$

if $y_0 = 0$ then $\lim_{t\to\infty} y(t) = 0$

if $y_0 < 0$ then $\lim_{t\to\infty} y(t) = 0$

Note: $y = 2$ is unstable but $y = 0$ is stable.