Math 222-006 (Prof. Young)

NAME:

Math 222 Quiz Feb 03, Spring 2016
Show all your work. No calculator.

Problem 1: Solve the initial value problem

$$y' = 2y^2 + xy^2, \quad y(0) = 1, \quad x > 0$$

and determine the behavior of the solution as $x \to \sqrt{6} - 2$.

$\frac{dy}{dx} = y^2(2+x)$, $\frac{dy}{y^2} = (2+x)\,dx$, $\int \frac{dy}{y^2} = \int (2+x)\,dx$, $-\frac{1}{y} = 2x + \frac{x^2}{2} + C$

$y(0)=1$, $-\frac{1}{1} = 2\cdot 0 + 0 + C$, $C = -1$, $-\frac{1}{y} = 2x + \frac{x^2}{2} - 1$, $y = \frac{-1}{2x + \frac{x^2}{2} - 1}$

$y = -\frac{2}{x^2+4x-2} = -\frac{2}{(x+2)^2 - 6}$ as $x \to \sqrt{6}-2$ from zero $y \to +\infty$

Problem 2: Solve the initial value problem

$$y' + \frac{1}{2}y = 2\cos t, \quad y(0) = -1.$$

$\mu' = \frac{1}{2}\mu$, $\mu = e^{t/2}$

$(e^{t/2}y)' = 2\cdot \cos t \cdot e^{t/2}$, $e^{t/2}y = 2\int \cos t \cdot e^{t/2}\,dt$

$\int \underset{u}{\cos t}\, \underset{v'}{e^{t/2}}\,dt = \cos t\, 2e^{t/2} - \int -\sin t\, 2e^{t/2}\,dt = 2\cos t\, e^{t/2} + 2\int \sin t\, e^{t/2}\,dt$

$= 2\cos t\, e^{t/2} + 2\cdot \sin t \cdot 2e^{t/2} - 4\int \cos t \cdot 2e^{t/2}\,dt$

$5\int \cos t\, e^{t/2}\,dt = 2\cos t\, e^{t/2} + 4\sin t\, e^{t/2} + C$

$e^{t/2}y = \frac{4}{5}\cos t\, e^{t/2} + \frac{8}{5}\sin t\, e^{t/2} + C$

$y = \frac{4}{5}\cos t + \frac{8}{5}\sin t + Ce^{-t/2}$

$y(0) = -1$, $= \frac{4}{5} + C$, $C = -\frac{9}{5}$

$y = \frac{4}{5}\cos t + \frac{8}{5}\sin t - \frac{9}{5}e^{-t/2}$