NAME: Solution

## Math 222 Quiz February 10, Spring 2016
### Show all your work. No calculator.

- Problem 1. A tank has capacity 6 L and initially contains 11 mg of salt dissolved in 3 L of water in the tank. A solution containing 1 mg/L of salt enters the tank at a rate of 3 L/day and the well-stirred mixture leaves the tank at a rate of 2 L/day.

  (a) Find the time when the tank is full.

  (b) Find the amount of salt (in milligrams) in the tank at any time before the tank is full. What is the amount of salt (in mg) in the tank when it is full and what is the concentration of the mixture then (in mg/L)?

- Problem 2. Consider the IVP $y' = ty$, $y(0) = 1$. Approximate the solution at $t = 0.3$ using Euler's method with a time step $h = 0.1$.

Prob 1 : $\dfrac{dV}{dt} = 3 - 2 = 1,$  $V = V_0 + t,$  $V_0 = 3,$  $\boxed{V = 3 + t}$

(a)  $V = 6$ when the tank is full, $3 + t = 6,$ $\boxed{t = 3 \text{ days}}$

(b)  $\dfrac{dQ}{dt} = 3 \cdot 1 - \dfrac{Q}{3+t} \cdot 2 = 3 - \dfrac{2}{3+t} Q$

$Q' + \dfrac{2}{3+t} Q = 3,$   $\mu' = \dfrac{2}{3+t} \mu,$  so  $\dfrac{\mu'}{\mu} = \dfrac{2}{3+t},$ $\ln \mu = 2 \ln(3+t)$

$\mu = (t+3)^2$   $\left[ (3+t)^2 Q \right]' = 3(t+3)^2,$  $(3+t)^2 Q = (t+3)^3 + C$

$Q(0) = 11,$   $3^2 Q(0) = 3^3 + C$   $C = 72$

$99 = 27 + C,$   $\boxed{Q = t+3 + \dfrac{72}{(t+3)^2}}$

$t = 3$ when tank is full  $Q(3) = 3+3 + \dfrac{72}{(3+3)^2} = 6 + 2 = 8 \text{ mg}$

concentration when tank is full $\Rightarrow \dfrac{8 \text{ mg}}{6 L} = \boxed{\dfrac{4}{3} \dfrac{mg}{L}}$

Prob 2 : $y_{n+1} = y_n + h \cdot f(t_n, y_n),$   $f(t_n, y_n) = t_n y_n$ in this problem.

$n=1,$ $t_1 = 0,$ $y_1 = 1$

$n=2,$ $t_2 = 0.1,$ $y_2 = y_1 + 0.1 \cdot t_1 y_1 = \boxed{y_2 = 1}$

$n=3,$ $t_3 = 0.2,$ $y_3 = y_2 + 0.1 \cdot t_2 y_2 = 1 + 0.1 \cdot 0.1 \cdot 1 = \boxed{1.01} = y_3$

$n=4,$ $t_4 = 0.3,$ $y_4 = y_3 + 0.1 \cdot t_3 y_3 = 1.02 + 0.1 \cdot 0.2 \cdot 1.01 = \boxed{1.0302} = y_4$