MATH222-004, QUIZ #2, February 1, 2016

NAME: Solution

1. (a) Classify the differential equation $y''' - 3y'' - 2y' + 2y = 0$.
   (b) Determine the values of r for which the differential equation has solutions of the form $y = e^{rt}$. Hint: one value is $r = -1$.

2. Solve the initial value problem

$$t^3 y' + 4t^2 y = e^{-t} \quad , \quad y(-1) = 0$$

#1| a| Linear, 3rd order.

b| plug $y = e^{rt}$ into the diff. eqn: $\underbrace{r^3 e^{rt}}_{y'''} - 3\underbrace{r^2 e^{rt}}_{y''} - 2\underbrace{r e^{rt}}_{y'} + 2\underbrace{e^{rt}}_{y} = 0 \Rightarrow$

$(r^3 - 3r^2 - 2r + 2) e^{rt} = 0 \Rightarrow r^3 - 3r^2 - 2r + 2 = 0$. One root is given as $r = -1$ so $r^3 - 3r^2 - 2r + 2 = (\ ?\ )(r+1)$. Find (?) by long division:

$$\begin{array}{l} r^3 - 3r^2 - 2r + 2 \ \lfloor \ r+1 \\ \underline{-r^3 - r^2} \qquad\qquad r^2 - 4r + 2 \\ 0 \ -4r^2 - 2r + 2 \\ \underline{+4r^2 + 4r} \\ \quad 2r + 2 \\ \quad \underline{-2r - 2} \\ \qquad 0 \end{array}$$

So, $r_1 = -1$, $r_{2,3} = \dfrac{4 \pm \sqrt{16 - 8}}{2} = 2 \pm \dfrac{\sqrt{8}}{2} = 2 \pm \sqrt{2}$

Thus $y_1(t) = e^{-t}$, $y_2(t) = e^{(2+\sqrt{2})t}$, and $y_3(t) = e^{(2-\sqrt{2})t}$

#2| $y' + \dfrac{4}{t} y = \dfrac{e^{-t}}{t^3}$, $\mu(t) = e^{\int p(t)dt} = e^{\int \frac{4}{t} dt} = e^{4 \ln t} = e^{\ln t^4} = t^4$

so

$y(t) = \dfrac{1}{t^4} \displaystyle\int^t t'^4 \frac{e^{-t'}}{t'^3} dt' + \frac{C}{t^4} = \frac{1}{t^4} \underbrace{\int^t t' e^{-t'} dt'}_{\text{do by parts}} + \frac{C}{t^4} =$

$\int t' e^{-t'} dt' = -t' e^{-t'} - \int (-e^{-t'}) dt' = -t' e^{-t'} - e^{-t'}$

$= \dfrac{1}{t^4}\left[-te^{-t} - e^{-t}\right] + \dfrac{C}{t^4}$, $y(-1) = 0 \Rightarrow e^{+1} - e^{+1} + C = 0 \Rightarrow C = 0$

$y(t) = \frac{1}{t^4}\left[-te^{-t} - e^{-t}\right]$