Math 222-006 (Prof. Young)

NAME: Solution

Math 222 Quiz Feb 29, Spring 2016
Show all your work. No calculator.

Problem 1: Use the method of undetermined coefficients to find the form (do not compute the coefficients) of a particular solution for the given equation

$$y'' + 3y' + 2y = e^t(t^2+1)\sin(2t) + 3e^{-t}\cos(t) + 4e^t.$$

corresponding homogeneous solution: $r^2+3r+2=0$, $(r+2)(r+1)=0$, $r=-2,-1 \Rightarrow y_1=e^{-2t}$, $y_2=e^{-t}$

$$Y = e^t(At^2+Bt+C)\sin 2t + e^t(Dt^2+Et+F)\cos 2t + (G\cos t + H\sin t)e^{-t} + Ie^t.$$

Problem 2: Given that $y_1(t) = t^{-1}$ is a solution of $2t^2y'' + 3ty' - y = 0$, $t > 0$. Use the method of reduction of order to find the other linearly independent fundamental solution y_2.

$y_2 = vy_1$, $y_2' = v'y_1 + vy_1'$, $y_2'' = v''y_1 + 2v'y_1' + vy_1''$

$2t^2(v''y_1 + 2v'y_1' + vy_1'') + 3t(v'y_1 + vy_1') - vy_1 = 0$

$2t^2(v''y_1 + 2v'y_1') + 3tv'y_1 = 0$

$2t^2(v''\cdot\frac{1}{t} - 2v'\frac{1}{t^2}) + 3tv'\frac{1}{t} = 0$, $2tv'' - 4v' + 3v' = 0$, $2tv'' = v'$, $\frac{v''}{v'} = \frac{1}{2t}$

$\ln v' = \frac{1}{2}\ln t$, $v' = \sqrt{t}$, $v = \frac{1}{3/2}t^{3/2}$, $\boxed{y_2 = t^{3/2}\cdot t^{-1} = t^{1/2}}$

Problem 3: Find the general solution of the given equation

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0.$$

$r^2+4r+4=0$, $r=-2,-2$ $y_1 = e^{-2t}$, $y_2 = te^{-2t}$. $W(y_1, y_2) = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t} - 2te^{-2t} \end{vmatrix} = e^{-4t}$

$$Y = \left(\int -\frac{t^{-2}e^{-2t}\cdot te^{-2t}}{e^{-4t}}\right)e^{-2t} + \left(\int \frac{t^{-2}e^{-2t}e^{-2t}}{e^{-4t}}\right)te^{-2t}$$

$$= \left(\int -\frac{1}{t}\right)e^{-2t} + \left(\int \frac{1}{t^2}\right)te^{-2t}$$

$$= -(\ln t)e^{-2t} - e^{-2t}$$

$$\boxed{y = c_1e^{-2t} + c_2te^{-2t} - \ln t\cdot e^{-2t}}$$