

Math 222 FINAL EXAM May 4, 2012

Read each problem carefully. Show all your work. No calculators!

1. Find the general solution of each of the following differential equations:

(a) (4)  $\frac{2}{x} \frac{dy}{dx} = y - 4$

(b) (4)  $y'' - 6y' + 9y = 0$

(c) (8)  $y'' + 3y' + 2y = 4x + 2$

2. (14) Solve the IVP:  $y'' - 4y' + 13y = 4\delta(t - 10)$ ;  $y(0) = y'(0) = 0$ .

3. (14) Express the solution of the following initial value problem in terms of a convolution integral:

$$y'' - 4y' + 5y = g(t); \quad y(0) = y'(0) = 0.$$

4. (14) Solve the IVP:  $X' = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} X$ ,  $X(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

5. (a) (10) Find the solution of the following boundary value problem, or show that the solution does not exist:

$$y'' + 9y = \sin x; \quad y(0) = 0, \quad y'(\pi) = 0$$

(b) (4) Find the least period of each of the following functions, or show that the function is not periodic:  $\cos 7x$ ;  $\tan 3x$ ;  $\sinh 2x$ ;  $\sin^2 x$

6. Consider the periodic function:  $f(x) = \begin{cases} 2 + x, & -2 \leq x < 0 \\ 2 - x, & 0 \leq x < 2 \end{cases}$   $f(x+4) = f(x)$

(a) (4) Sketch the graph of the above function for three periods and state whether the function is odd, even, or neither.

(b) (10) Find the Fourier series for the above function.

7. Consider the function:  $f(x) = \pi - x$ ,  $0 \leq x < \pi$ .

(a) (4) Sketch three periods of the odd  $2\pi$ -periodic extensions of  $f(x)$ .

(b) (10) Find the Fourier series for the odd  $2\pi$ -periodic extensions of  $f(x)$ .

MATH 222 EXAM III

Nov. 18, 2015

Read each problem carefully. Show all your work for each problem. No Calculators!

1. (18 pts) Seek a power series solution of form  $y = \sum_{n=0}^{\infty} a_n x^n$  for the following equation

$$y'' + xy' + 2y = 0.$$

- (a) Find the recurrence relation.  
 (b) Find the first 3 terms of each of two linearly-independent solutions,  $y_1$  and  $y_2$ .

2. (14 pts) Solve the following initial value problem

$$x^2 y'' - 3xy' + 4y = 0, \quad y(1) = 1, \quad y'(1) = 0.$$

3. (14 pts) Sketch the function

$$f(t) = \begin{cases} 0 & 0 \leq t < 1 \\ 2 + t & 1 \leq t < 2 \\ 5 & t \geq 2 \end{cases}$$

and find its Laplace transform using only the definition of the Laplace Transform.

4. (16 pts) Find the Inverse Laplace transform of

$$(a) F(s) = \frac{e^{-3s}(s-2)}{s^2 + 4s + 8}, \quad (b) F(s) = \frac{s^2 - 4s + 6}{(s-2)^3}$$

5. (13 pts) Solve the following initial value problem:

$$y'' + 2y' + y = u_1(t)e^{-(t-1)} \cos(t-1), \quad y(0) = 0, \quad y'(0) = 0.$$

6. (13 pts) Solve the following initial value problem:

$$y'' + y' - 6y = \delta(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 1.$$

7. (12 pts) Express the solution of the given initial value problem in terms of a convolution integral.

$$y'' + y' - 6y = g(t), \quad y(0) = 0, \quad y'(0) = 0.$$

Notice that the left-hand side of the differential equation is identical to that of Problem 6.