Review Material on Calculus for Differential Equations Math 222

1 Derivative

- Let $g(x) = \cos(f(x))$, find the derivative of $g(x)$ if $f'(x) = x^2$.
- What is $h'(x)$ if $h(x) = \sin(x)x^2$?
- If $z'(x) = \frac{1}{3+5x^2}$, what is $z(x)$?
- If $qq' = \frac{1}{\sqrt{x^2+1}}$, what is $q(x)$?
- Let $f(t) = te^{at} + b$, where $a \neq 0$ and b are constant coefficients and $t > 0$. (a) Find a and b such that $f(t)$ has a critical point at $t = 2$. (b) Continue from (a), find b such that $f = 3$ at the critical point. (c) Finally, determine the behavior of $f(t)$ as $t \to \infty$ with a and b from above. (d) With a and b from (b), sketch $f(t)$ for $t \in [0, 10]$.
- Let $g(t) = \sin te^{\alpha t} + \beta$, with $\alpha \neq 0$ and β constant coefficients. Find the conditions on α and β such that there is more than one critical point for $g(t)$. Find the expression for the critical points in terms of α and β.
- Let $y(t) = 5e^{-\frac{t}{10}}\cos(\omega_0 t)$ for $t > 0$. (a) Sketch $y(t)$ for $t \in [0, \frac{10\pi}{\omega_0}]$ if the oscillation frequency $\omega_0 = 2\pi$ per second (Hertz). (b) If $y(4) = 0$, what can one say about the frequency ω_0? (c) The function $y(t)$ has a time-varying oscillation amplitude $5e^{-\frac{t}{10}}$. Find the time T when this time-dependent amplitude first decreases to less than 1.
- Compute the Taylor series of $g(x) = \cos(x)$ around $x = 0$.
- Compute the Taylor series of $h(x) = \sin(x)$ around $x = 0$.
- Let x and y satisfy the equation $x^2 + 4y^2 = 4$. Find $\frac{dy}{dx}$ =? Find $\frac{dx}{dy}$ =?

2 Integration

- Find $\int \frac{1}{1-x^2}dx$ =?
- Find $\int \frac{1}{\sqrt{1-x^2}}dx$ =?
- Use integration by parts to find $\int xe^{-x}dx$ =?
- What is $\int \cos(x)xdx$?
- Find $\int \frac{xdx}{1+x^4}$.

- Use the Taylor series for $\cos(x)$ and $\sin(x)$ above to show that $\cos'(x) = -\sin(x)$ and $\sin'(x) = \cos(x)$, or equivalently $\int \cos(x)dx = \sin(x) + c$ and $\int -\sin(x)dx = \cos(x) + c$.
- If $F(x) = \int \frac{1}{e^x+1}dx$, what is $F'(x)$?
- What is the integral $\int_{-\infty}^{\infty} \frac{1}{1-x^2}dx$?
- Use the identity $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$ to compute the integral $\int \frac{1}{1-x}dx$. Verify your answer against the Taylor series of $\ln(1-x)$ around zero.
- Compute the integral $\int_0^{\infty} \cos(x)e^{-sx}dx$ with $s > 0$ a constant.