Solution for Extra HW Week 11

Problem 1 (Problem 13 p355)

$y''+\omega^2 y = g,\ y(0)=0,\ y'(0)=1$

$\mathcal{L}[y''+\omega^2 y] = \mathcal{L}[g] = G$

$s^2Y - sy(0) - y'(0) + \omega^2 Y = G,\quad (s^2+\omega^2)Y = 1+G,$

$Y = \frac{1}{s^2+\omega^2} + \frac{G}{s^2+\omega^2},\quad y(t) = \frac{1}{\omega}\sin\omega t + \frac{1}{\omega}\int_0^t \sin\omega(t-\tau)g(\tau)d\tau$

Problem 2 (problem 20 p355)

$y^{(4)} + 5y'' + 4y = g,\quad y(0)=1,\ y'(0)=0,\ y''(0)=0,\ y'''(0)=0$

$\mathcal{L}[y^{(4)}+5y''+4y] = \mathcal{L}[g] = G$

$s^4Y - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0) + 5(s^2Y - sy(0) - y'(0)) + 4Y = G$

$(s^4+5s^2+4)Y = G + s^3 + 5s,\quad Y = \frac{G}{s^4+5s^2+4} + \frac{s^3+5s}{s^4+5s^2+4}$

$Y = \frac{G}{(s^2+4)(s^2+1)} + \frac{s^3+5s}{(s^2+4)(s^2+1)}$

$H = \frac{1}{(s^2+4)(s^2+1)} = \frac{1}{3}\left(\frac{1}{s^2+1} - \frac{1}{s^2+4}\right),\quad \mathcal{L}[H] = \frac{1}{3}\cdot\sin t - \frac{1}{6}\sin 2t \equiv h(t)$

$J \equiv \frac{s(s^2+5)}{(s^2+4)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1},\quad (As+B)(s^2+1) + (Cs+D)(s^2+4) = s^3+5s$

$s^3:\ A + C = 1$

$s^2:\ B + D = 0$

$s^1:\ A + 4C = 5 \qquad C = \frac{4}{3},\ A = -\frac{1}{3}$

$s^0:\ B + 4D = 0 \qquad B = 0 = D$

$J = \frac{-\frac{1}{3}s + 0}{s^2+4} + \frac{\frac{4}{3}s}{s^2+1},\quad \mathcal{L}^{-1}[J] = -\frac{1}{3}\cos 2t + \frac{4}{3}\cos t$

$\therefore\ y(t) = \mathcal{L}^{-1}\left[\frac{G}{(s^2+4)(s^2+1)} + \frac{s^3+5s}{(s^2+4)(s^2+1)}\right] = \int_0^t h(t-\tau)g(\tau)d\tau - \frac{1}{3}\cos 2t + \frac{4}{3}\cos t$

Problem 3 (problem 22 p367)

(a) $V_1' = 1.5 + 1.5 - 3.0 = 0,\quad V_1 = 30$ gal

$V_2' = 1 + 3 - 4.0 = 0,\quad V_2 = 20$ gal

$\frac{dQ_1}{dt} = 1.5\times 1 - 3\cdot\frac{Q_1}{30} + 1.5\cdot\frac{Q_2}{20},$

$\frac{dQ_2}{dt} = 3\times\frac{Q_1}{30} - \frac{Q_2}{20}\times 4$

(b) $1.5 - \frac{Q_1}{10} + \frac{3Q_2}{40} = 0$

$\frac{Q_1}{10} - \frac{Q_2}{5} = 0$

$1.5 - \frac{Q_2}{5} + \frac{3Q_2}{40} = 0 \quad \frac{3}{2} = \frac{5}{40}Q_2,\ Q_2 = \underline{12}$ oz

$Q_1 = 2\cdot Q_2 = 24$ oz

(c) $x_1 = Q_1 - Q_1^E = Q_1 - 24,\quad x_2 = Q_2 - Q_2^E = Q_2 - 12$

$x_1' = 1.5 - \frac{1}{10}(x_1+24) + \frac{3}{40}(x_2+12) = -\frac{1}{10}x_1 + \frac{3}{40}x_2$

$x_2' = \frac{1}{10}(x_1+24) - \frac{1}{5}(x_2+12) = \frac{x_1}{10} - \frac{x_2}{5}$