Math 222, Spring 2017.

Present your work in an organized fashion. Make sure that your work is algebraically correct and logically sound. Show all your work. No calculator, notes, or books.

In-class Quiz, on 02/13/2017 for M222-008

1. Draw a direction field for the differential equation

$$y' = y(y-2)^2.$$

   Describe the dependence of the solution on the initial condition.

2. Solve the initial value problem

$$ty' + 2y = 4t^2, \quad y(1) = y_0.$$

   Sketch the solution when $y_0 = 1$. Sketch another solution when $y_0 = 2$.

3. Solve the IVP

$$y'' + 2y' - 3y = 0, \quad y(0) = \alpha, \quad y'(0) = \beta.$$

   Find conditions on α and β such that the solution approaches zero in the limit $t \to \infty$.

4. Consider the IVP

$$y' = 2t, \quad y(0) = 1.$$

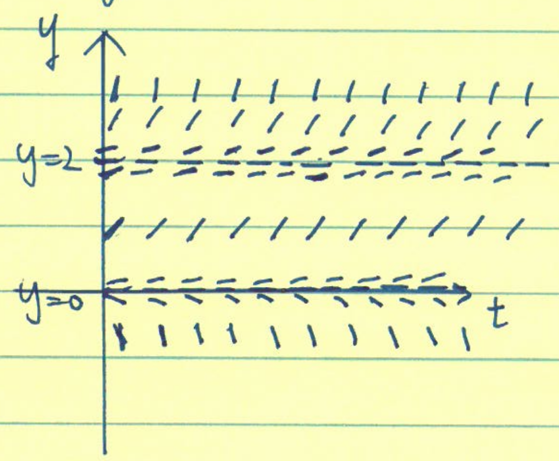
   Use Euler's method with a time step size $h = 0.1$ to approximate the solution at $t = 0.3$. Compare against the exact solution evaluated at the same time $t = 0.3$ by computing the difference between the approximate solution and the exact solution.

P.1

QUIZ 02/13/2017

Problem 1: $y' = y(y-2)^2$

efuilibrium $y' = 0 = y(y-2)^2$, $y = 0, 2, 2$



$y' > 0$

$y' > 0$

$y' < 0$

with an initial condition $y_0 > 2$, $y \to \infty$ as $t \to \infty$

$0 < y_0 < 2$ $y \to 2$ as $t \to \infty$

$y_0 < 0$ $y \to -\infty$ as $t \to \infty$

Problem 2 $ty' + 2y = 4t^2$, $y(1) = y_0$

$y' + \frac{2}{t}y = 4t$, $\mu = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

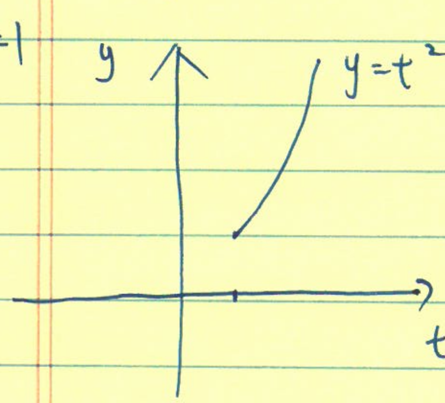
$y = \dfrac{\int 4t \cdot t^2 dt + c}{t^2} = \dfrac{t^4 + c}{t^2} = t^2 + \dfrac{c}{t^2}$

$y(1) = y_0 = 1 + c$, $c = y_0 - 1$, $y = t^2 + \frac{y_0 - 1}{t^2}$

$y_0 = 1$, $c = 0$ $y = t^2$

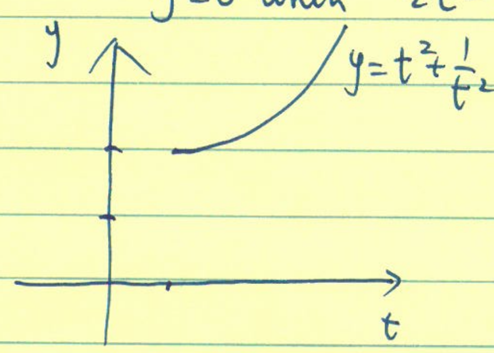
$y_0 = 2$, $c = 1$. $y = t^2 + \frac{1}{t^2}$

$y_0 = 1$



$y_0 = 2$, $y = t^2 + \frac{1}{t^2}$, $y' = 2t - \frac{2}{t^3}$

$y' = 0$ when $2t - \frac{2}{t^3} = 0$, $t = 1$



Problem 3

$y''+2y'-3y=0, \quad y(0)=\alpha, \quad y'(0)=\beta$

$r^2+2r-3=0$ $\qquad y=c_1e^{-3t}+c_2e^{t}$

$(r+3)(r-1)=0,$

$r=-3, 1$ $\qquad$ want $y \to 0$ as $t \to \infty$

$\Rightarrow$ therefore $c_2=0$

$y=c_1e^{-3t}, \quad y'=-3c_1e^{-3t}$

$y(0)=c_1=\alpha$

$y'(0)=-3c_1=\beta \Rightarrow \boxed{-3\alpha=\beta}$

Problem 4

$y'=2t, \quad y(0)=1$

$y=t^2+c_1 \quad y(0)=1=c_1$

$y=t^2+1$

$y(0.3)=0.3^2+1$

$=1.09$

difference between solution at $t=0.3$ & Euler's approximation at $t=0.3$ $=1.09-1.06$

$=0.03$

$\longrightarrow y_{n+1}=y_n+h\cdot 2t_n$

$y_0=1$ at $t_0=0$

with $h=0.1$

$t_1=t_0+h=0.1$

$y_1=y_0+h\cdot 2t_0$

$y_1=1+0.1\times 2\times 0=1$

$y_2=y_1+2\cdot t_1\cdot h$

$=1+2\times 0.1\times 0.1$

$y_2=1+0.02=1.02$

at $t=t_3=t_2+h=0.3$

$y_3=y_2+2\cdot t_2\times h$

$=1.02+2\times 0.2\times 0.1$

$=1.02+0.04$

$=1.06$