Math 222, Spring 2017.

Present your work in an organized fashion. Make sure that your work is algebraically correct and logically sound. Show all your work. No calculator, notes, or books.

In-class Quiz, on 03/31/2017 for M222-008

1. Use Laplace transform to solve the IVP

$$y'' - y' - 2y = 0,\ y(0) = 1,\ y'(0) = 0.$$

2. Sketch the graph of the given function, express $f(t)$ in terms of the unit step function $u_c(t)$, and then find the Laplace transform of $f(t)$.

(a)

$$f(t) = \begin{cases} 0, & t < 2 \\ (t-3)^2, & t \geq 2 \end{cases}$$

(b)

$$f(t) = \begin{cases} 0, & t < \pi \\ t - \pi, & \pi \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$$

3. Find the inverse Laplace transform of the given function

(a)

$$F(s) = \frac{2(s-1)e^{-2s}}{s^2 + 2s + 2},$$

(b)

$$F(s) = \frac{e^{-4s}}{2s^2 + 1}$$

(c)

$$F(s) = \frac{1}{(s+1)(s^2-4)}$$

Solution to QUIZ 03/31

Problem 1 : $y'' - y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$

$$\mathcal{L}[y'' - y' - 2y] = 0$$

$$s^2 Y - y'(0) - s y(0) - (sY - y(0)) - 2Y = 0$$

$$(s^2 - s - 2) Y = s - 1$$

$$Y = \frac{s-1}{s^2 - s - 2} = \left(\frac{A}{s-2} + \frac{B}{s+1}\right), \quad A(s+1) + B(s-2) = s - 1$$

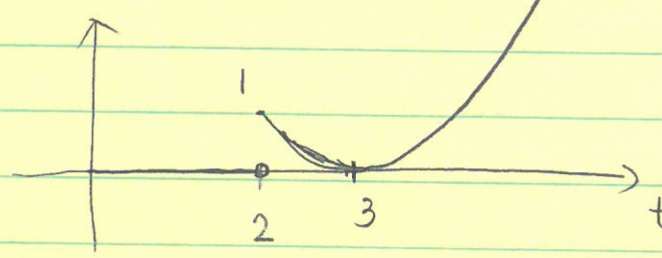
$$-3B = -2 \quad B = 2/3$$

$$3A = 1, \quad A = 1/3$$

$$Y = \frac{1/3}{s-2} + \frac{2/3}{s+1}$$

$$y = \frac{1}{3} e^{2t} + \frac{2}{3} e^{-t}$$

Problem 2 : (a) $f(t) = \begin{cases} 0 & t < 2 \\ (t-3)^2 & t \geq 2 \end{cases}$



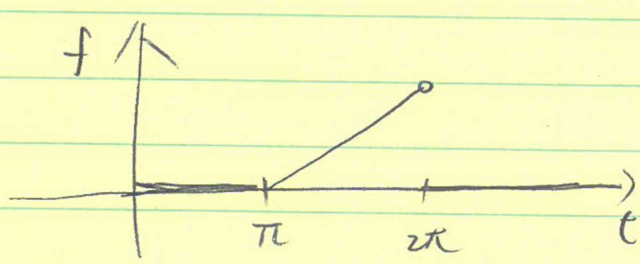
$$f(t) = (t-3)^2 u_2$$

$$f(t) = (t-2-1)^2 u_2 = \left((t-2)^2 - 2(t-2) + 1\right) u_2$$

$$\mathcal{L}[f] = \mathcal{L}\left[(t-2)^2 u_2 - 2(t-2) u_2 + u_2\right]$$

$$= \frac{2}{s^3} e^{-2s} - \frac{2}{s^2} e^{-2s} + \frac{1}{s} e^{-2s}$$

(b) $f(t) = \begin{cases} 0 & t < \pi \\ t - \pi & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}$



$$f(t) = (t-\pi) u_\pi + (\pi - t) u_{2\pi}$$

$$\mathcal{L}[f] = \mathcal{L}\left[(t-\pi) u_\pi - (t - 2\pi + \pi) u_{2\pi}\right]$$

$$= \frac{e^{-\pi s}}{s^2} - \frac{e^{-2\pi s}}{s^2} - \frac{\pi e^{-2\pi s}}{s}$$

Problem 3 (a) $F = \dfrac{2(s-1)e^{-2s}}{s^2+2s+2}$, $F = 2\dfrac{s-1}{s^2+2s+1+1}\cdot e^{-2s}$

$$F = 2\frac{s+1-2}{(s+1)^2+1}e^{-2s}$$

$$\mathcal{L}^{-1}\left[\frac{s+1-2}{(s+1)^2+1}\right] = e^{-t}.\cos t - 2\cdot e^{-t}\sin t$$

$$\mathcal{L}^{-1}\left[2\cdot\frac{s+1-2}{(s+1)^2+1}e^{-2s}\right] = 2\left[e^{-(t-2)}\cos(t-2) - 2e^{-(t-2)}\sin(t-2)\right]u_2$$

(b) $F(s) = \dfrac{e^{-4s}}{2s^2+1} = \dfrac{1}{2}\dfrac{1}{s^2+\left(\frac{1}{\sqrt{2}}\right)^2}e^{-4s}$

$$\mathcal{L}^{-1}[F] = \frac{1}{2}\sqrt{2}\sin\left(\frac{t-4}{\sqrt{2}}\right) \quad u_4$$

(c) $F(s) = \dfrac{1}{(s+1)(s^2-4)} = \dfrac{1}{(s+1)(s+2)(s-2)} = \dfrac{A}{s-2}+\dfrac{B}{s+1}+\dfrac{C}{s+2}$

$$A(s+1)(s+2)+B(s-2)(s+2)+C(s-2)(s+1)=1$$

$s=-1$ $\quad B\cdot(-3)(1)=1$, $B=-1/3$

$s=-2$ $\quad C(-4)(-1)=1$, $C=\frac{1}{4}$

$s=2$ $\quad A\cdot 3\cdot 4 = 1$, $A=\frac{1}{12}$

$$\mathcal{L}^{-1}\left[\frac{1}{12}\frac{1}{s-2} - \frac{1}{3}\frac{1}{s+1} + \frac{1}{4}\frac{1}{s+2}\right]$$

$$= \frac{1}{12}e^{2t} - \frac{1}{3}e^{-t} + \frac{1}{4}e^{-2t}$$