QUIZ 09/26

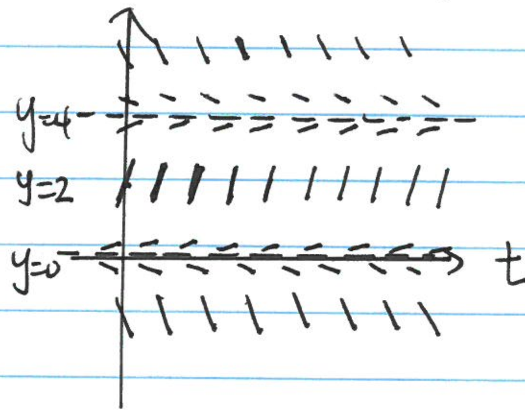
Problem 1: $y' = y(4-y)$

$y' = 0 = y(4-y)$ at equilibrium $y = 0, 4$

$y > 4$, $y' < 0$

$0 < y < 4$, $y' > 0$

$y < 0$, $y' < 0$



$y = 2$, $y' = 2 \cdot 2 = 4$

if $y(0) < 0$, $y' < 0$ for all time, $y \to -\infty$ as $t \to \infty$

$0 < y(0) < 4$, $y' > 0$, $y \to 4$ as $t \to \infty$

$4 < y(0)$, $y' < 0$ $y \to 4$ as $t \to \infty$

Problem 2: $y' = ay + b$ because $y \to 2$ as $t \to \infty$

$y' = 0$ at $y = 2$, $\cancel{2y} \; 2a + b = 0$, $\boxed{a = -b/2}$

also for $y \to 2$ as $t \to \infty$, y must decrease to 2 if $y(0) > 2$

y must increase to 2 if $y(0) < 2$

$\Rightarrow y' < 0$ if $y(0) > 2$ $\quad a\cancel{y_0}y + b < 0$ if $y(0) > 2$

$y' > 0$ if $y(0) < 2$ $\quad a(\cancel{y_0}y - 2) < 0$ if $y(0) > 2$

this means $\boxed{a < 0}$

Problem 3: $ty' + (t+1)y = t$, $y(\ln 2) = 1$

$y' + \left(1 + \frac{1}{t}\right)y = 1$, $\mu = e^{\int 1 + \frac{1}{t} dt} = e^{t + \ln t} = te^t$

$y = \dfrac{\int te^t dt + c}{te^t}$, $\int te^t dt = te^t - \int e^t dt = te^t - e^t$

$$y = \frac{te^t - e^t + c}{te^t} = 1 - \frac{1}{t} + \frac{c}{t}e^{-t}$$

$$y(\ln 2) = 1 = 1 - \frac{1}{\ln 2} + \frac{c}{\ln 2}e^{-\ln 2} = 1 - \frac{1}{\ln 2} + \frac{c}{2\ln 2}$$

$$-1 + \frac{1}{\ln 2} = \frac{c}{2\ln 2}, \quad \boxed{c = -2\ln 2 + 2}$$

$$y = 1 - \frac{1}{t} + (-2\ln 2 + 2)\frac{e^{-t}}{t}$$

$$\boxed{\text{as } t \to \infty, \ y \to 1}$$

Problem 4: $\frac{dy}{dx} = \frac{2x}{y + x^2 y}$, $y(0) = 2$

$$\frac{dy}{dx} = \frac{2x}{y(1+x^2)}, \qquad y\,dy = \frac{2x}{1+x^2}dx$$

$$\int y\,dy = \int \frac{2x}{1+x^2}dx = \int \frac{du}{u} \qquad u = 1+x^2, \ du = 2x\,dx$$

$$\frac{y^2}{2} = \ln|u| + c = \ln|1+x^2| + c = \ln(1+x^2) + c$$

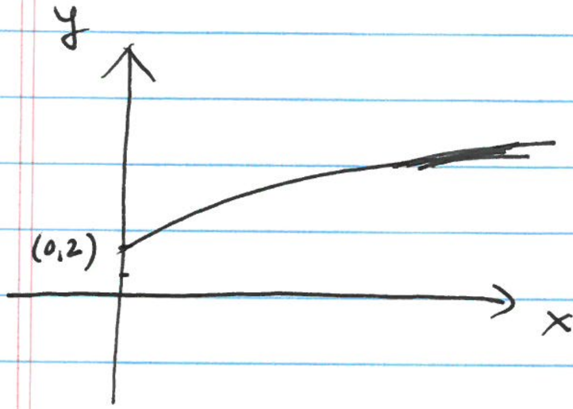
$$y(0) = 2 = \frac{4}{2} = \ln(1) + c, \quad c = 2$$

$$\frac{y^2}{2} = \ln(1+x^2) + 2, \quad y^2 = 2\ln(1+x^2) + 4,$$

$$y = \pm\sqrt{2\ln(1+x^2) + 4}, \quad y(0) = 2 \Rightarrow y = +\sqrt{2\ln(1+x^2) + 4}$$

$$\lim_{x\to\infty} \sqrt{2\ln(1+x^2) + 4} = \lim_{x\to\infty} \sqrt{4\ln x} = \infty$$

P.03



Problem 5: $r_1 = 2,\ r_2 = -3,\quad (r-2)(r+3)=0,\quad r^2+r-6=0$

$\boxed{y''+y'-6y=0}$ or $2y''+2y'-12y=0$ ... etc.

Problem 6: $r^2-r-2=0\quad (r-2)(r+1)=0,\quad r=-1,2$

$y = c_1e^{-t}+c_2e^{2t}\qquad y(0)=\alpha = c_1+c_2$

$y' = -c_1e^{-t}+2c_2e^{2t}\qquad y'(0) = -c_1+2c_2 = 2$

want α such that $y\to 0$ as $t\to\infty$

$\Rightarrow\quad c_2 = 0$

$\Rightarrow\quad \alpha = c_1+0 \Rightarrow \alpha = c_1$

$-c_1+2c_2 = -c_1+2\cdot 0 = 2,\qquad c_1=-2$

$\Rightarrow \boxed{\alpha=-2}$